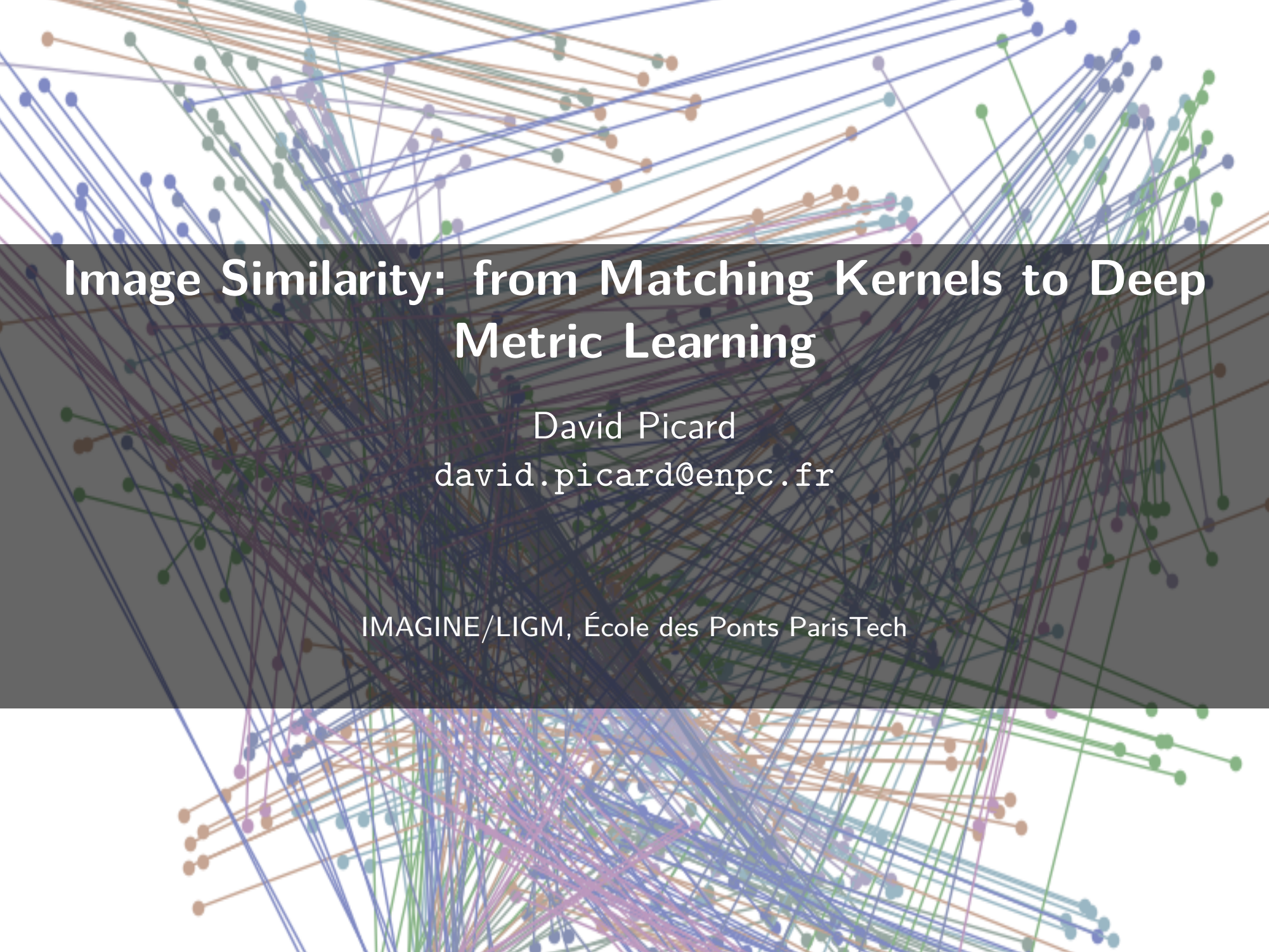

Image Similarity: from Matching Kernels to Deep Metric Learning

David Picard
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IMAGINE/LIGM, École des Ponts ParisTech

Outline

EMD

$$\min \sum w_{ij} d(x_i, x_j)$$

$$w_{ij} \geq 0, \sum_i w_{ij} = 1, \sum_j w_{ij} = 1$$

Wasserstein

MMD

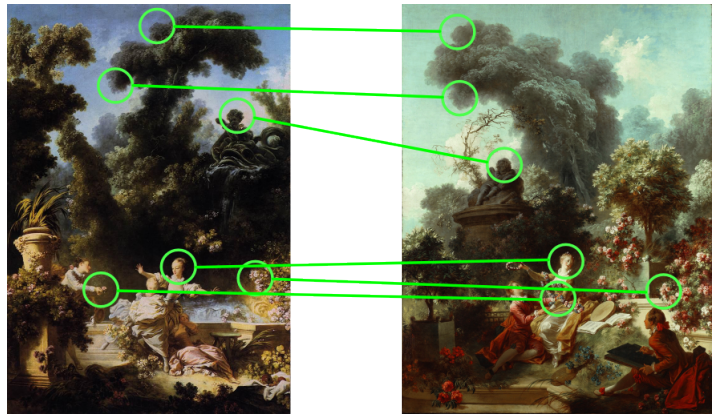
$$\sup_T \mathbb{E}_p[T(x)] - \mathbb{E}_q[T(x)]$$

HORDE $\mathbb{E}_{x \sim \mathcal{J}}[x^{\otimes k}]$

Matching Kernels

$(\approx \langle \phi(B_i), \phi(B_j) \rangle)$

Keypoints matching



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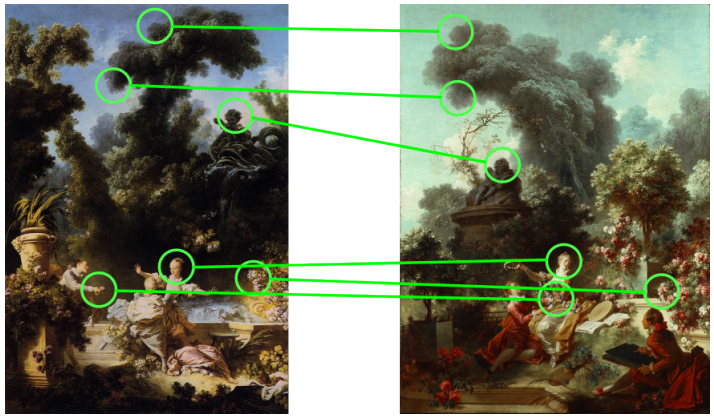
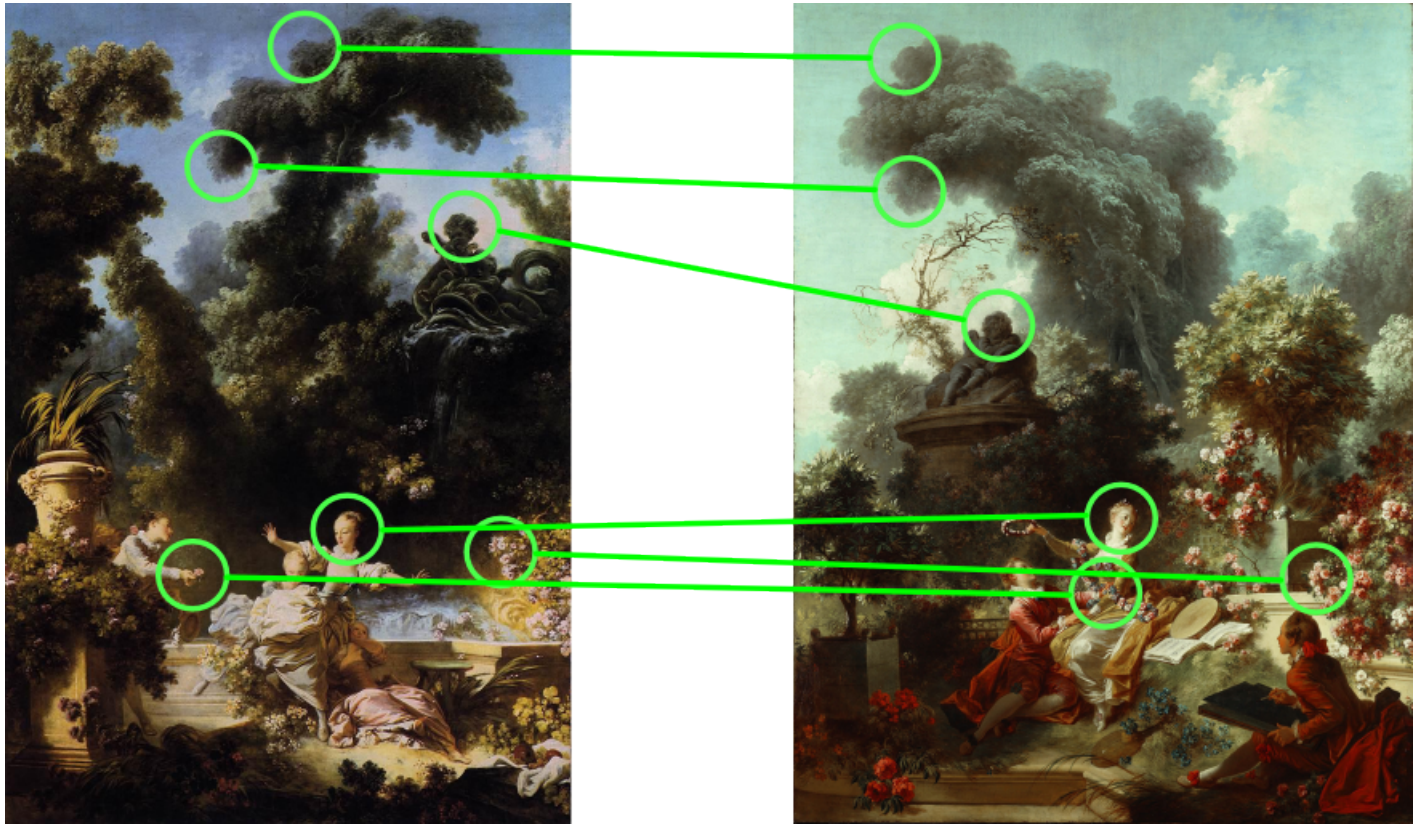


Image similarity using keypoints

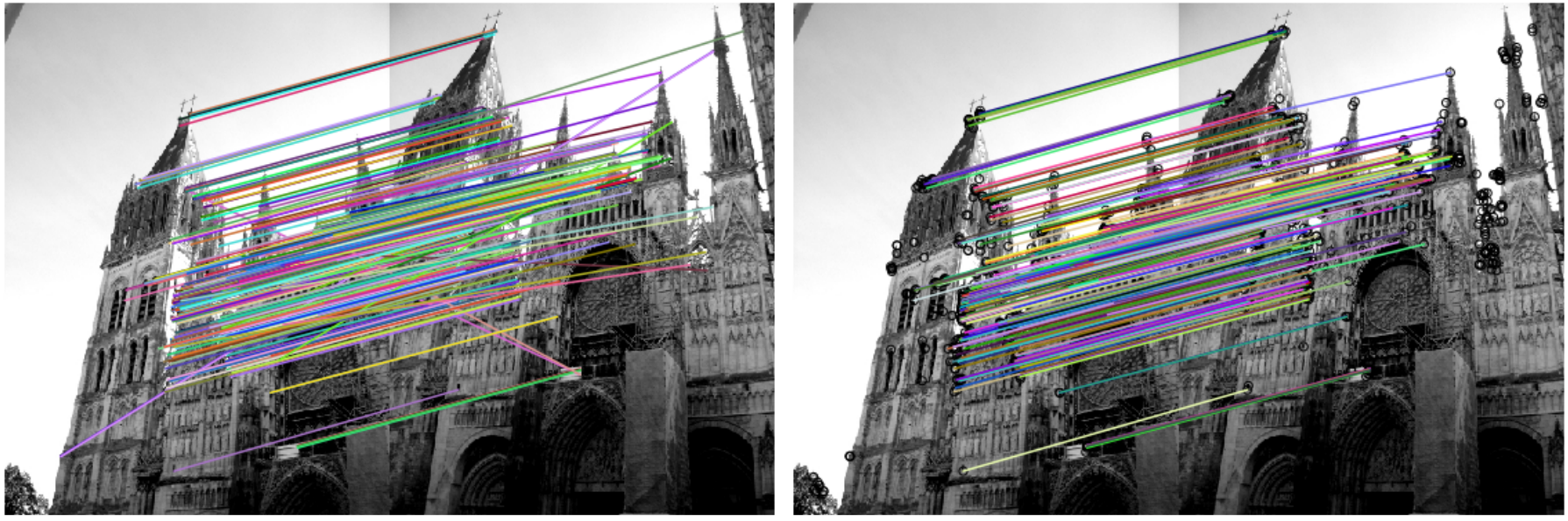
Principle: Visualy similar image ⇒ Many similar regions



- Find informative regions
- Describe informative regions (map to a vector space)
- Count number of matching regions (*e.g.*, 1-NN + distance threshold)

Geometric consistency

Keypoint matching: reconstruction methods ⇒ Preserving geometry



Use consensus algorithm to remove outliers (*e.g.*, ransac)

Generic image similarity

- Different views → different scenes
- Local geometric consistency (*e.g.*, inside an object)

Keypoint matching limitations

Keypoints matching does not correspond to an inner product

- Not symmetric

Limited use for machine learning

- No intrinsic representation
- No classifier, regressor, etc

Limited use for downstream tasks (detection, recognition)

Caveat: Recent deep neural networks approaches to learn correspondances. *E.g.*, S2DNet: Learning Accurate Correspondences for Sparse-to-Dense Feature Matching. Hugo Germain, Guillaume Bourmaud, and Vincent Lepetit. In Proceedings of the European Conference on Computer Vision (ECCV), 2020.

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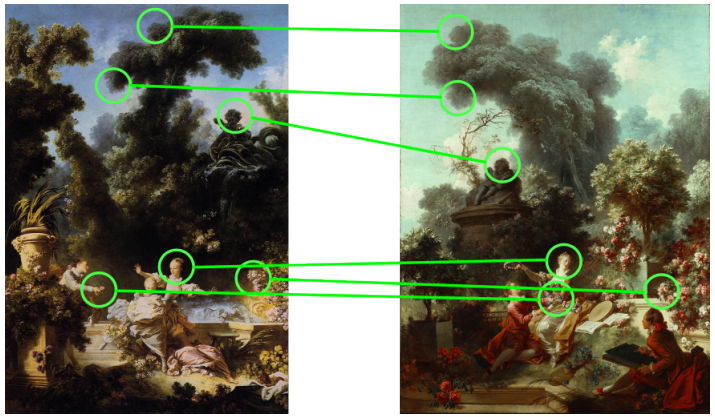
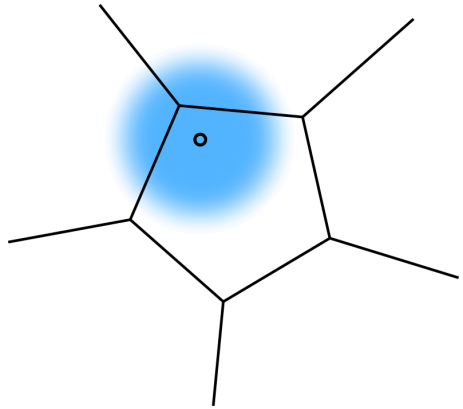


Kernel on bags

Image := unordered set of descriptors $B_i = \{x_{ri}\}$

$\Rightarrow$ Similarity on unordered sets (**bags**)

Kernel on bags



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Define similarity between descriptors (**minor kernel**):

- ex: dot product $k(x_{ri}, x_{sj}) = \langle x_{ri}, x_{sj} \rangle$
- ex: Gaussian kernel : $k(x_{ri}, x_{sj}) = \exp(-\gamma \| x_{ri} - x_{sj} \|^2)$
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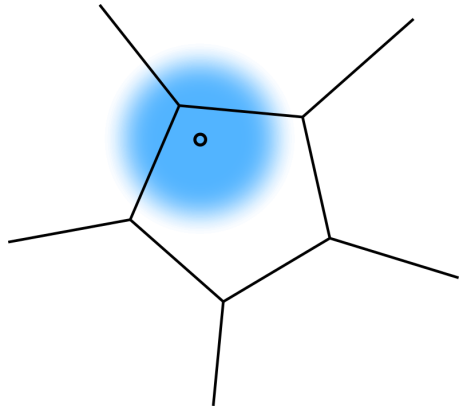

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Kernel on Bags: **kernel combination**

- Sum Kernel (Shawe-Taylor 2004)

$$K(B_i, B_j) = \sum_{x_{ri} \in B_i} \sum_{x_{sj} \in B_j} k(x_{ri}, x_{sj})$$

$$K(B_i, B_j) = \left\langle \sum_{x_{ri} \in B_i} \phi(x_{ri}), \sum_{x_{sj} \in B_j} \phi(x_{sj}) \right\rangle$$

Quantization kernel

Dictionary (codebook) of prototypical descriptors $\mathscr{D} = \{\mu_c\}_{1 \leq c \leq M}$

- k-means clustering over many descriptors
- Centroïds $\mu_c := \mathbf{codewords}$

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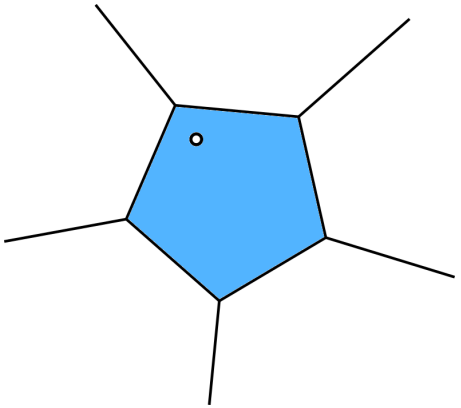
Quantization function h:

- $h_m(x_{ri}) = \begin{cases} 1 \text{ if } m = \arg\min_c \|x_{ri} - \mu_c\|^2, \\ 0 \text{ else} \end{cases}$
- **1-hot assignment** to clusters ($\in \mathbb{R}^M$)



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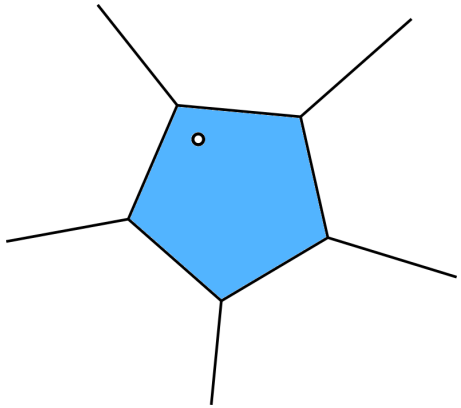

Bag of Visual Words:

$$K(B_i, B_j) = \sum_{x_{ri} \in B_i} \sum_{x_{sj} \in B_j} \sum_m h_m(x_{ri}) h_m(x_{sj})$$

$$= \langle \phi_q(B_i), \phi_q(B_j) \rangle, \quad \phi_q(B) = \sum_{x_r \in B} [h_m(x_r)]_m$$

- Histogram of occurrences of codewords (Sivic 03, Csurka 04)

Need for kernel linearization

Kernel on bags: ML compatible alternative to descriptor matching

- Implicit definition of an **image representation space**
- Choice of minor kernel ⇒ combination (sum, product)

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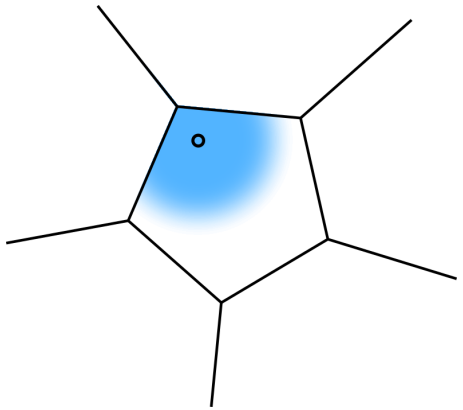

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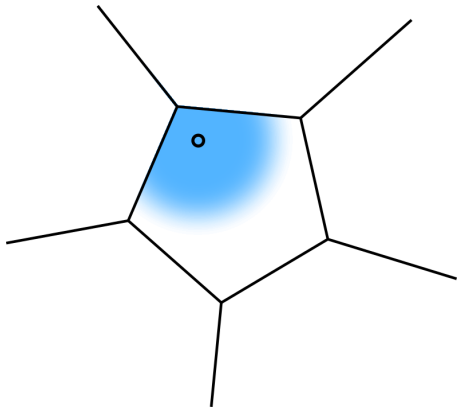


Computational cost:

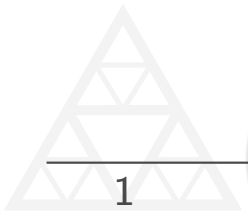
- Quadratic in number of descriptors
- Non-linear (costly) inner loop

We have to linearize → Explicit image representation space

Vector of Locally Aggregated Tensors

Gaussian-quantization kernel linearization:

- Assume $\|x_{ri}\| = 1$, then $k_g(x_{ri}, x_{sj}) \propto \exp(\gamma \langle x_{ri}, x_{sj} \rangle)$



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- $\exp(\gamma \langle x_{ri}, x_{sj} \rangle) = \sum_p \frac{\gamma^p \langle x_{ri}, x_{sj} \rangle^p}{p!} = \sum_p \frac{\gamma^p}{p!} \langle x_{ri}^{\otimes p}, x_{sj}^{\otimes p} \rangle$
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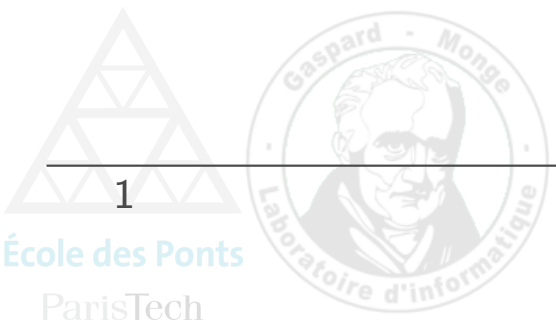

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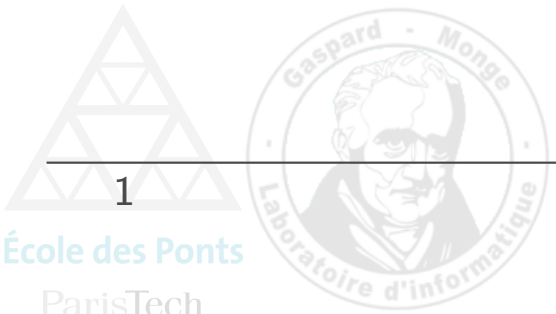

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1

Vector of Locally Aggregated Tensors[1]

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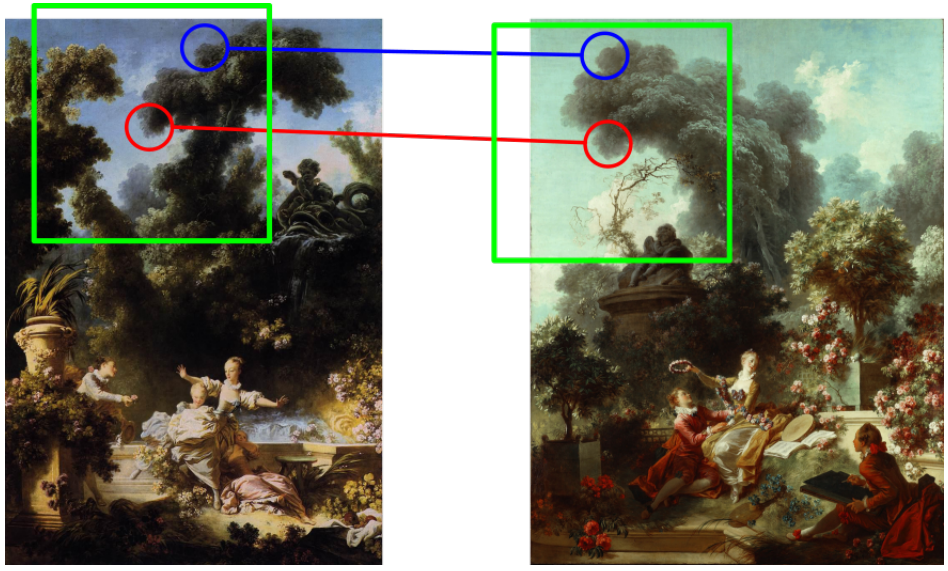
Explicit image representation → Keypoint matching similarity

[1] Picard, David, and Philippe-Henri Gosselin. "Improving image similarity with vectors of locally aggregated tensors." 2011 18th IEEE International Conference on Image Processing. IEEE, 2011.

Geometric consistency kernel

Embedding local geometric consistency in a kernel

⇒**Similar descriptors** should have **similar neighborhood**



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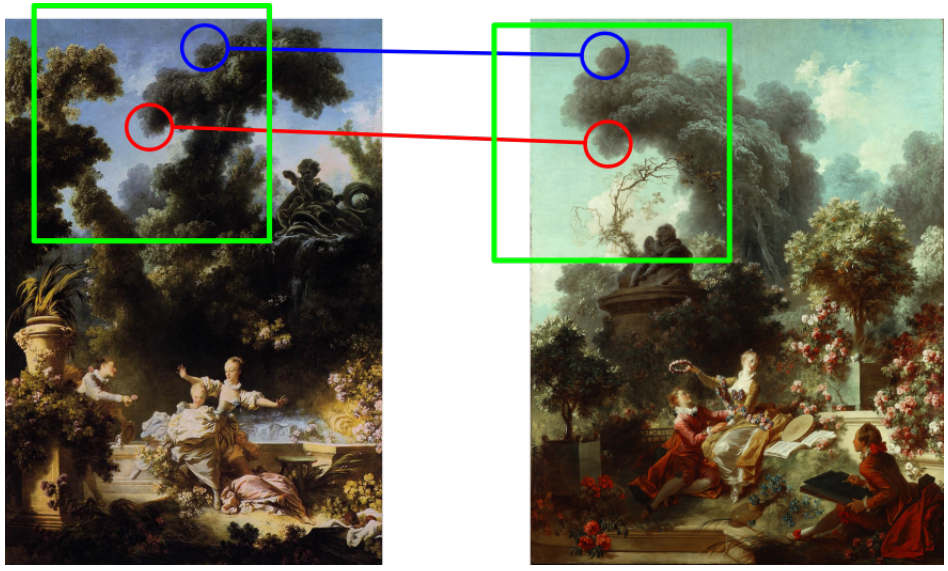

Nested matching kernels:

- $\Omega(x_{ri})$: Neighborhood of x_{ri}
- $k(x_{ri}, x_{sj}) = k_s(x_{ri}, x_{sj}) \sum_{u \in \Omega(x_{ri})} \sum_{v \in \Omega(x_{sj})} k_s(x_u, x_v)$

Ω defines the geometric properties (invariances)

Spatial tensor aggregation

Geometric consistency kernel linearization

- $k_s(x_{ri}, x_{sj}) = \langle \phi_s(x_{ri}), \phi_s(x_{sj}) \rangle$
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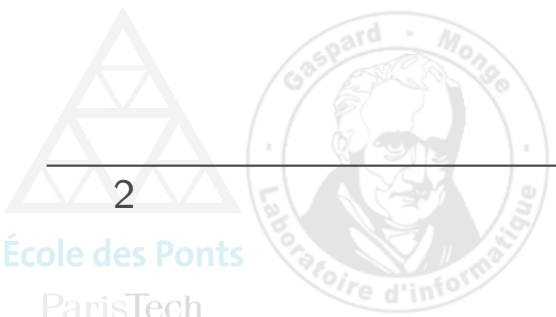

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Sum kernel:

- Linearization $\rightarrow$ Global mapping
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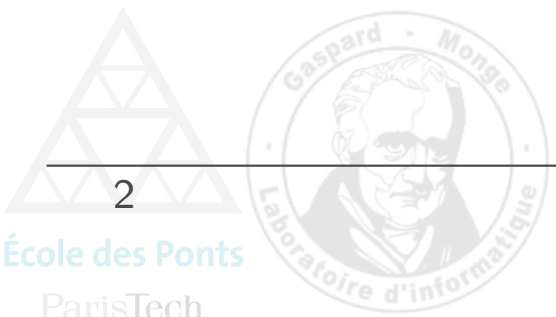

Spatial tensor aggregation [2]

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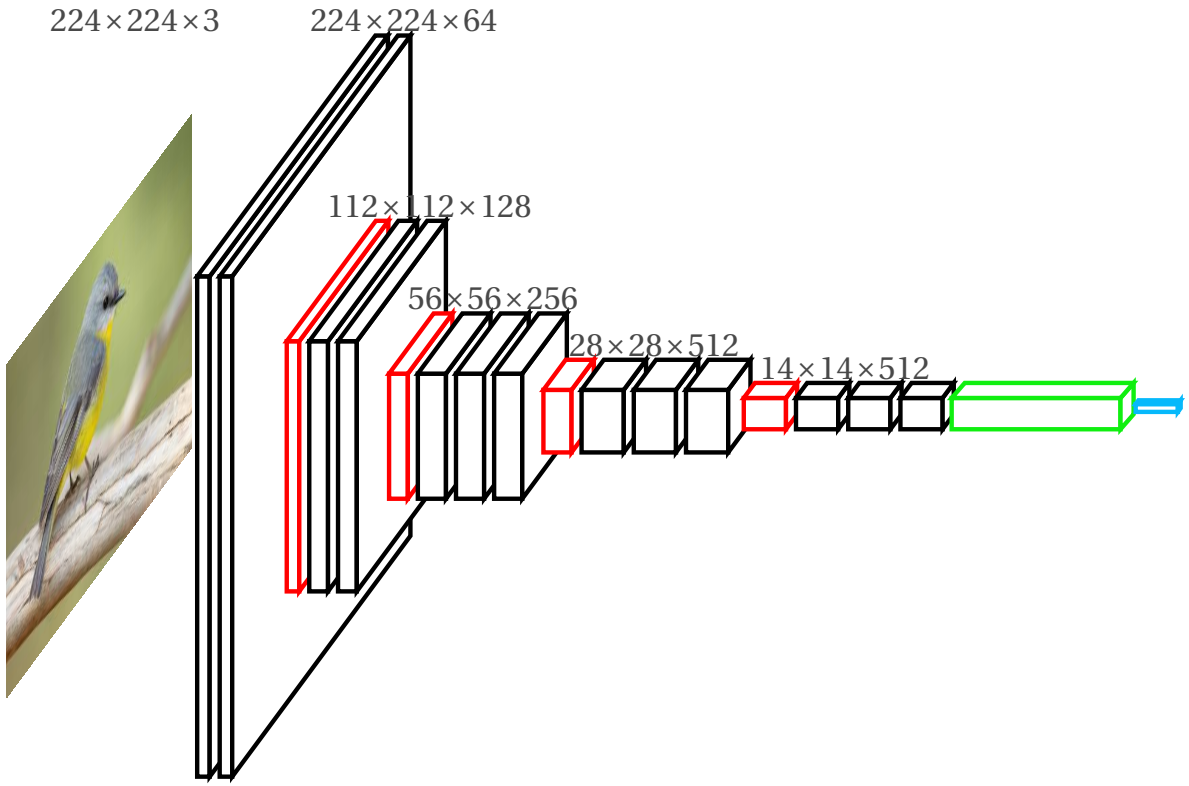
4th order tensor $\rightarrow$ Locally geometric consistent matching kernel

[2] Picard, David. "Preserving local spatial information in image similarity using tensor aggregation of local features." 2016 IEEE International Conference on Image Processing (ICIP). IEEE, 2016.

Deep Learning

Features from deep neural network



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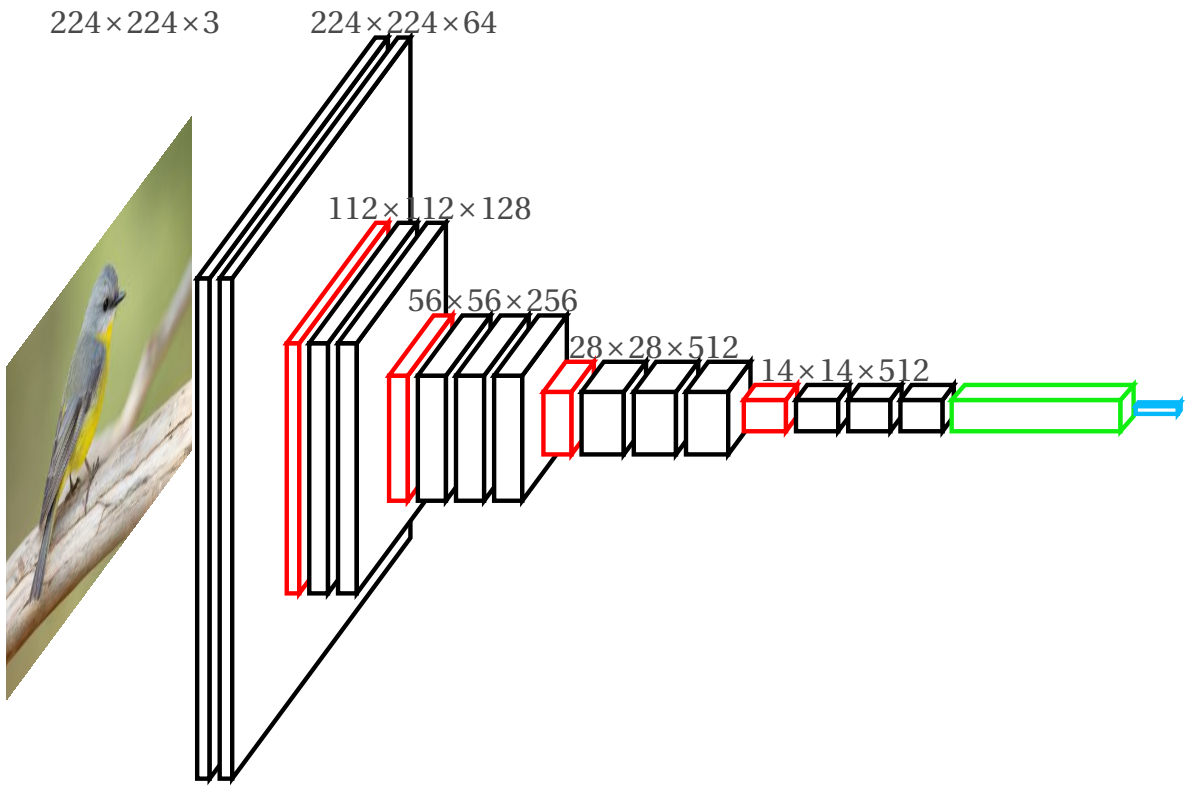

- **Local feature** extractor (CNN)

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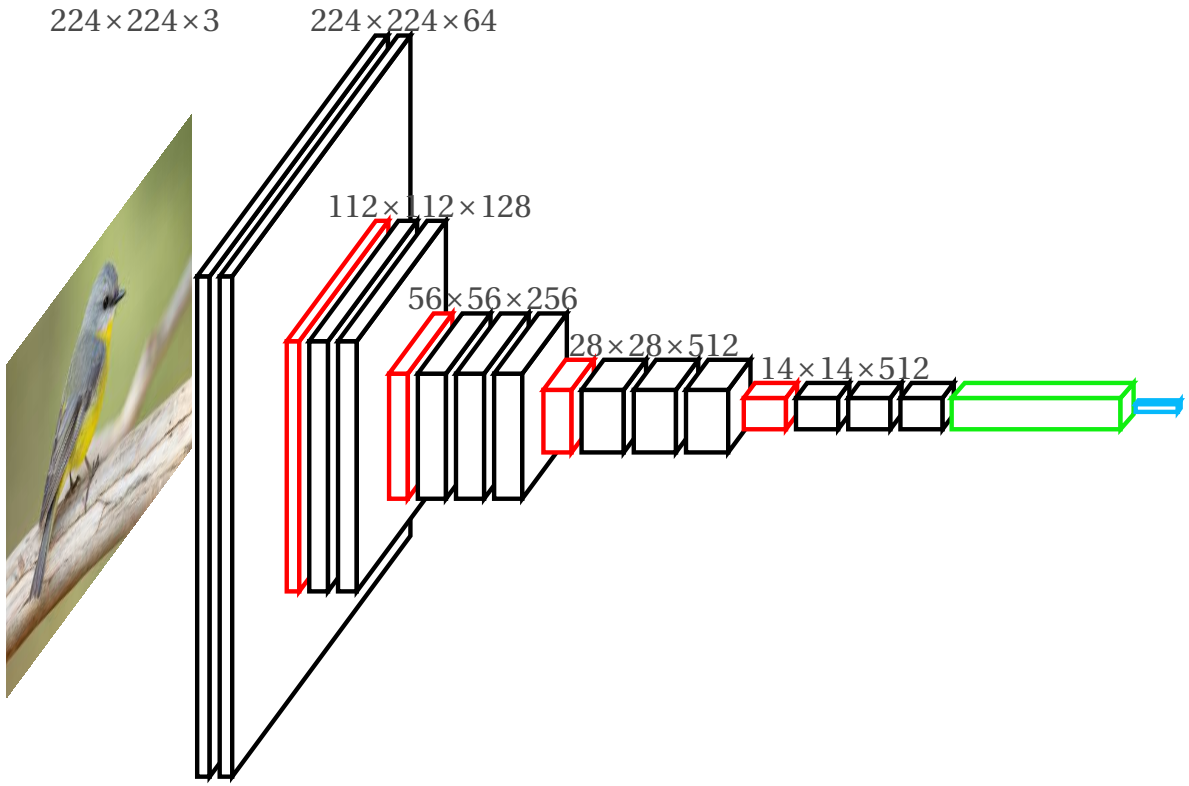

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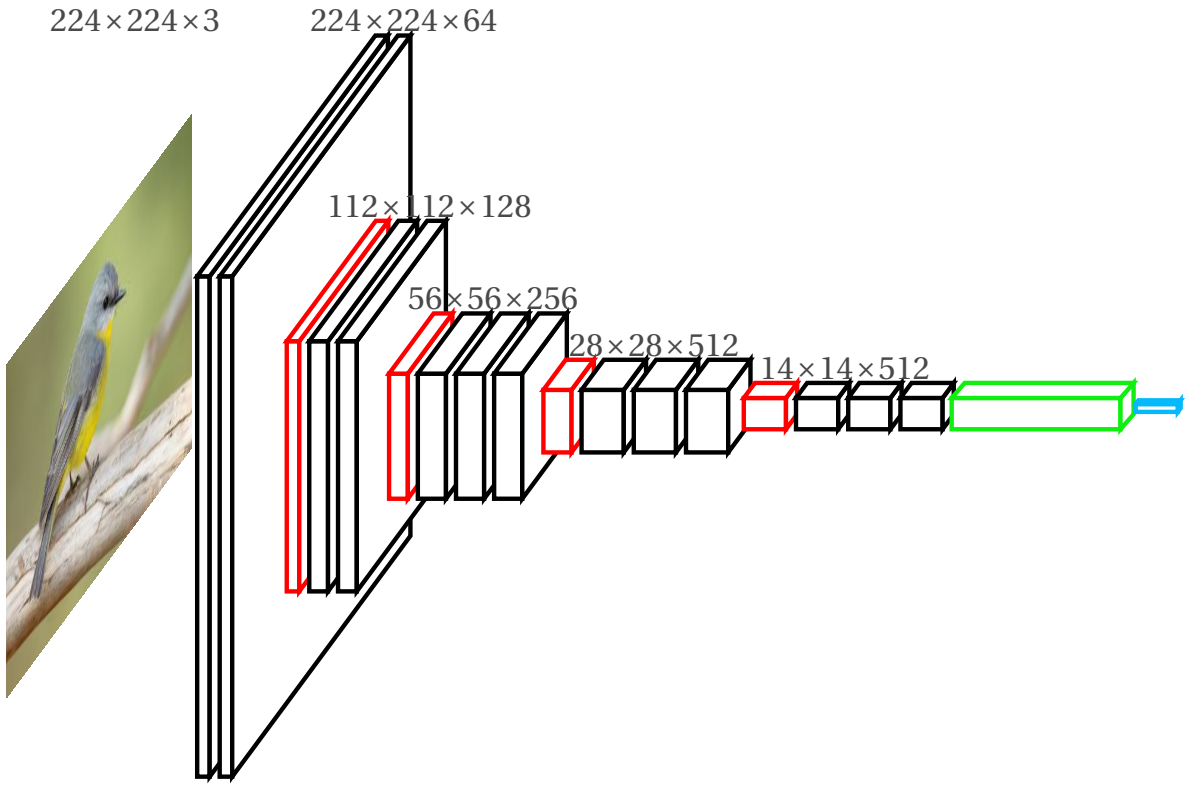

- **Local feature** extractor (CNN)
- Mapping to a vector space (*e.g.*, linearized matching kernel)
- End-to-end **metric learning** $\left(\min \mathbb{E}[\max(0, \langle x_q, x_n \rangle + \alpha - \langle x_q, x_p \rangle)]\right)$

End-to-end training?

Adapting **dictionary based matching kernels** to **deep learning**

End-to-end training?

Adapting **dictionary based matching kernels** to **deep learning**

- Learning discriminative dictionary (**backprop on** $\mathscr{D}$)

End-to-end training?

Adapting **dictionary based matching kernels** to **deep learning**

- Learning discriminative dictionary (**backprop on** $\mathscr{D}$)
- One hot encoding $\rightarrow$ differentiable assignement (**softmax**)

$$h_m(x) = \frac{\exp(\gamma \langle x, \mu_m \rangle)}{\sum_k \exp(\gamma \langle x, \mu_k \rangle)}$$

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End-to-end training?

Adapting **dictionary based matching kernels** to **deep learning**

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- Dictionary second order layer

$$y = h(x) \otimes x \otimes h(x) \otimes x$$

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High dimensional output $\rightarrow$ factorization

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Efficient Codebook and Factorization

Compression by projection

- $z_i(x) = \langle p_i, h(x) \otimes x \otimes h(x) \otimes x \rangle,\ p_i \in \mathbb{R}^{M \times d \times M \times d}$
- $z(x) = [z_i(x)]_{1 \leq i \leq D}$

Efficient Codebook and Factorization

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Compression by rank-1 factorization

- $\langle p_i, h(x) \otimes x \otimes h(x) \otimes x \rangle = \langle U_i, h(x) \otimes x \rangle \langle V_i, h(x) \otimes x \rangle$
- $z_i(x) = (h(x)^\top U_i x) \circ (h(x)^\top V_i x)$
- $U_i, V_i :=$ Mapping of x to the assignment space

Efficient Codebook and Factorization

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Compression by sharing

- Common pool of projectors $\tilde{U}, \tilde{V}$
- Combination of projectors $U_i = A_i \tilde{U}, V_i = B_i \tilde{V}$
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Efficient Codebook and Factorization [3]

Compression by projection

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Efficient codebook/2nd order repr. $\rightarrow$ **standard deep learning operators**

[3] Jacob, Pierre, et al. "Efficient codebook and factorization for second order representation learning." 2019 IEEE International Conference on Image Processing (ICIP). IEEE, 2019.

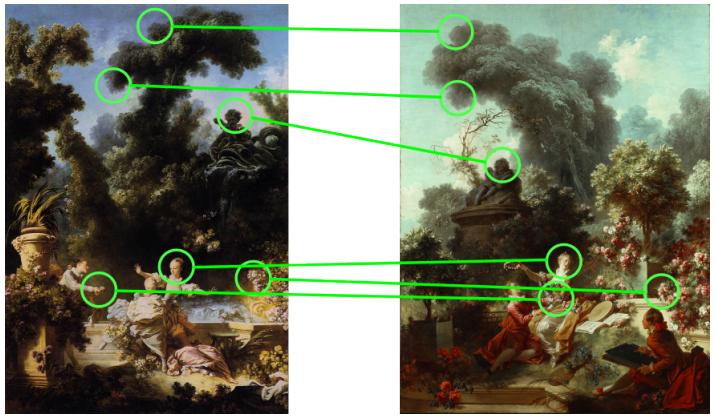
Outline

EMD

$$\min \sum w_{ij} d(x_i, x_j)$$

$$w_{ij} \geq 0, \sum_i w_{ij} = 1, \sum_j w_{ij} = 1$$

Wasserstein



MMD

$$\sup_T \mathbb{E}_p[T(x)] - \mathbb{E}_q[T(x)]$$

HORDE $\mathbb{E}_{x \sim \mathscr{I}}[x^{\otimes k}]$

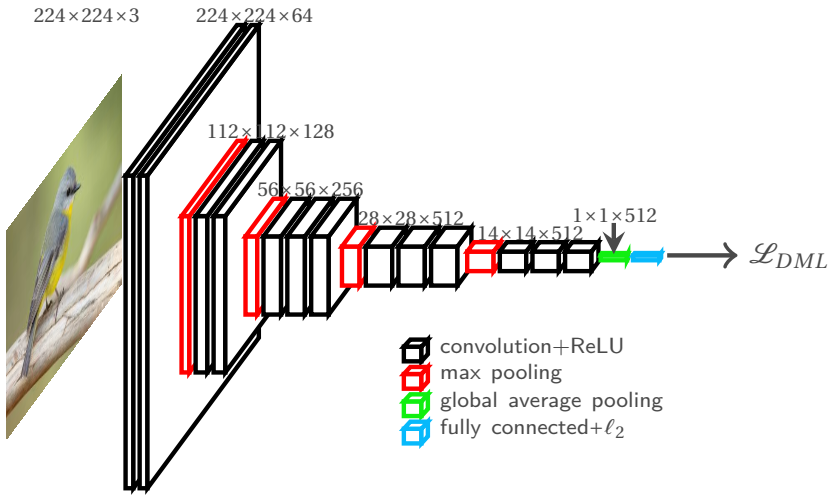
Matching Kernels

$$(\approx \langle \phi(B_i), \phi(B_j) \rangle)$$

Keypoints matching

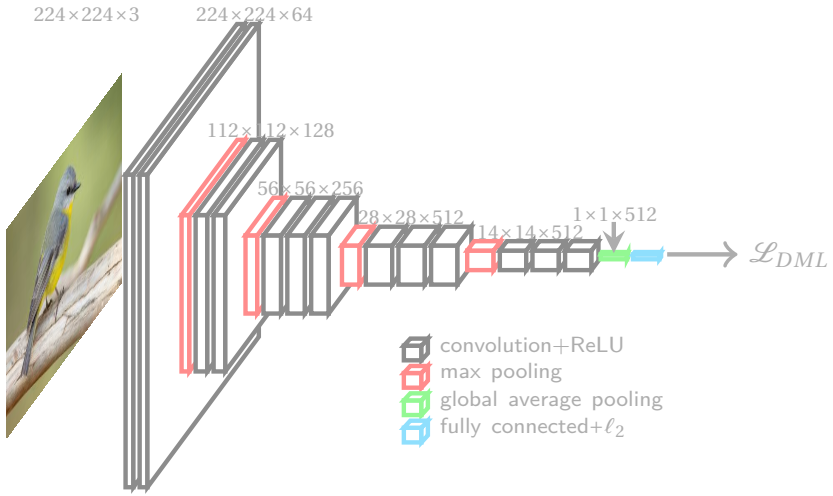
Optimizing the mean

Sum Kernel → Spatial averaging of all features



Optimizing the mean

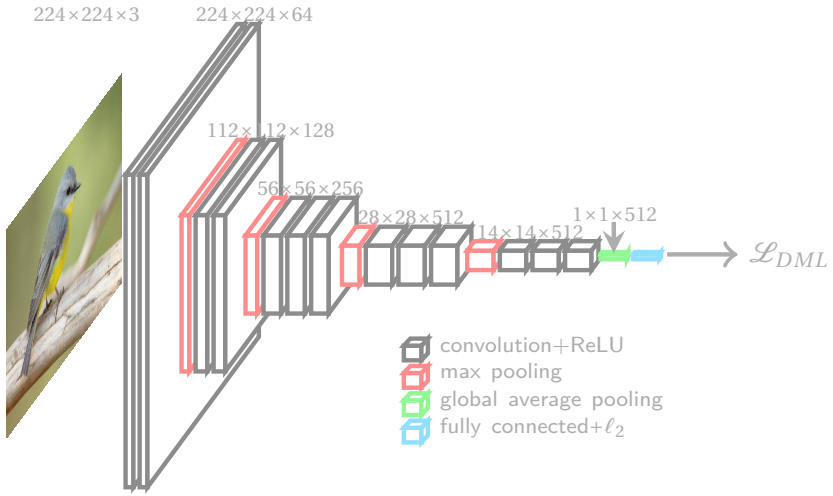
Sum Kernel → Spatial averaging of all features



- Learning similar/dissimilar averages

Optimizing the mean

Sum Kernel → Spatial averaging of all features



- Learning similar/dissimilar averages
- Ignoring the distribution of local features

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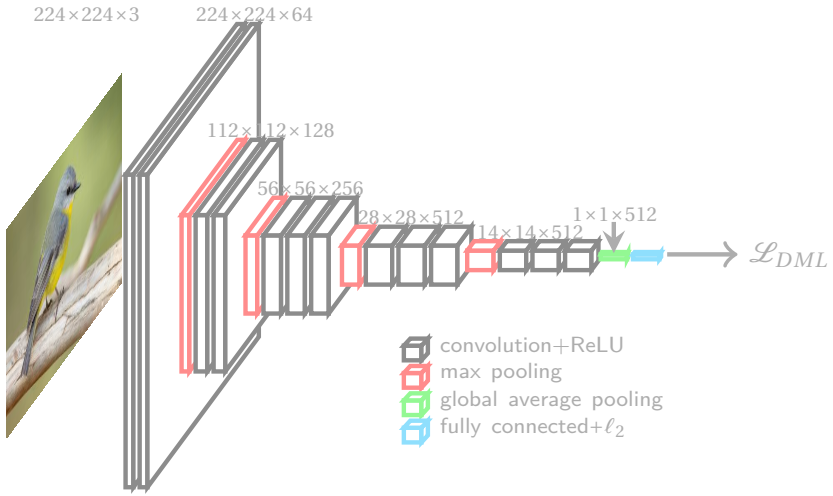
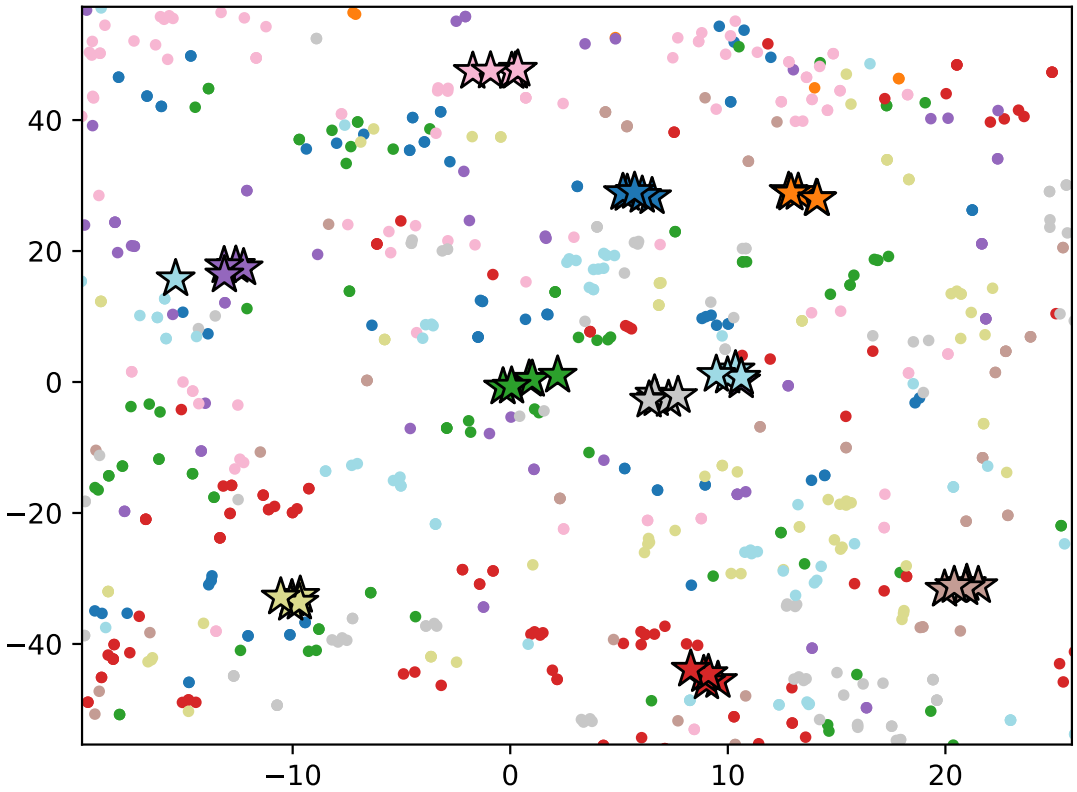


- Learning similar/dissimilar averages
- Ignoring the distribution of local features

Similar image → Similar local features???

Scattering problem

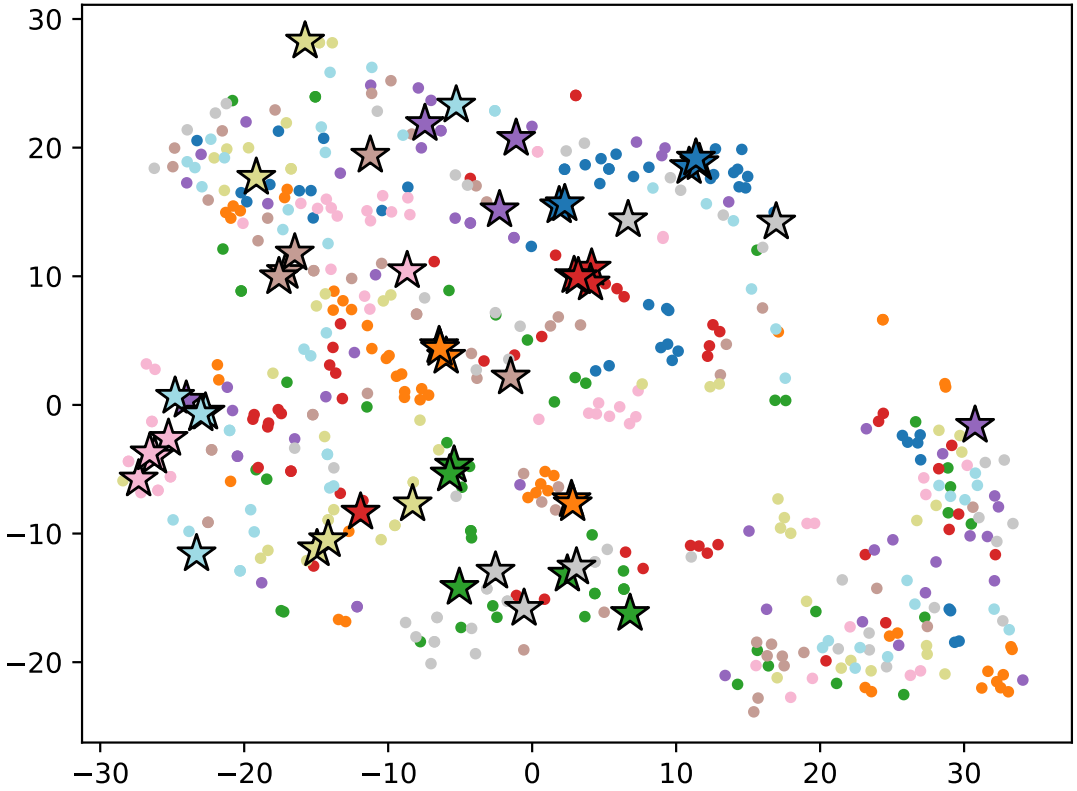
- local features, ⋆ image representations



- Good representations, Non-discriminative local features

Scattering problem

- local features, ⋆ image representations



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- Representations **not robust to sampling issues**

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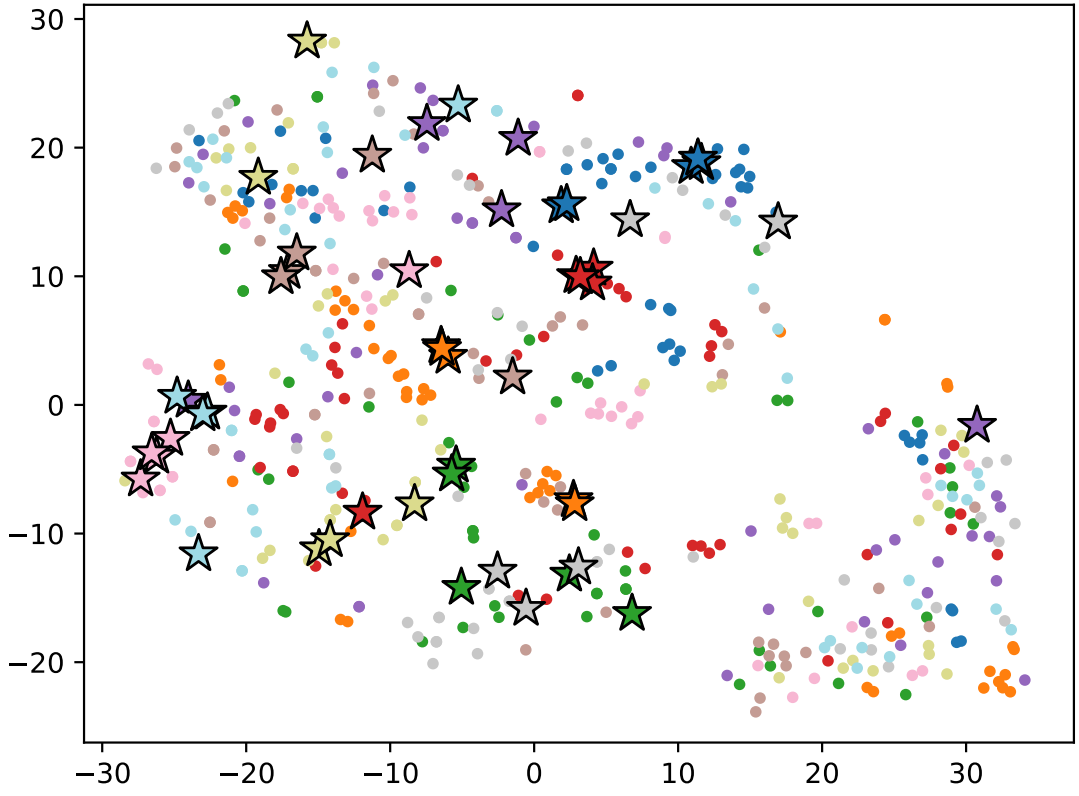

- Good representations, Non-discriminative local features
- Representations **not robust to sampling issues**

Distribution metric learning

Similar/dissimilar images have similar/dissimilar deep feature distributions

Tackling deep feature distributions

DML optimizes the mean (GAP):

$$\min_{\phi} \sum_{I,J \in \mathscr{P}} \|\mathbb{E}_{i\sim I}[\phi(x_i)] - \mathbb{E}_{j\sim J}[\phi(x_j)]\|^2 + \sum_{I,J \in \mathscr{N}} \max(0, \alpha - \|\mathbb{E}_{i\sim I}[\phi(x_i)] - \mathbb{E}_{j\sim J}[\phi(x_j)]\|^2)$$

⇒ **Highly different deep feature distributions with the same mean!**

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Tackling deep feature distributions

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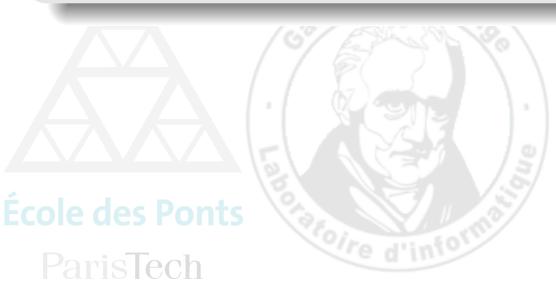
⇒ **Highly different deep feature distributions with the same mean!**

Distribution metric learning

We should perform metric learning such that:

- Similar images have similar deep feature distributions
- Dissimilar images have dissimilar deep feature distributions



Distributions of local features

Point set optimal assignment: EMD

$$\min_{w} \sum_{rs} w_{rs} d(\mathbf{x}_{ri}, \mathbf{x}_{sj})$$

$$\text{s.t.} \forall r, \forall s, w_{rs} \geq 0$$

$$\forall r, \sum_{s} w_{rs} = 1; \forall s, \sum_{r} w_{rs} = 1$$

Distributions of local features

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$$\forall r, \sum_{s} w_{rs} = 1; \forall s, \sum_{r} w_{rs} = 1$$

Random local features sampled from image distribution $\mathbf{x}_{ri} \sim \mathscr{D}_i$

- Distance between $\mathscr{D}_i$ and $\mathscr{D}_j$
- Wasserstein distance: minimum change to $\mathscr{D}_i$ to obtain local features that look as if sampled from $\mathscr{D}_j$

$$\inf \mathbb{E}[d(\mathbf{x}_{ri}, \mathbf{x}_{sj})]$$

Unknown moving deep feature distribution

Deep features distribution:

- **Unknown** (no model, few assumptions)
- **Changing** (opimized through gradient descent)

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Metric learning over distributions:

- Distance between distributions (Wasserstein, MMD)
- Kernel mean embedding $\leftrightarrow$ Maximum mean Discrepancy

$$\underbrace{\sup_{\|f\| \leq 1} \int f(x) d\mathbb{P}(x) - \int f(y) d\mathbb{Q}(y)}_{MMD} = \underbrace{\|\mu_{\mathbb{P}} - \mu_{\mathbb{Q}}\|_{\mathcal{H}}}_{\text{Kernel mean embedding}}$$

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Kernel Mean Embedding:

- $\mu_{\mathbb{P}} = \mathbb{E}_{\mathbb{P}}[\phi(x)]$, Gaussian kernel $\rightarrow \phi(x) \approx [x, x \otimes x, \ldots, x^{\otimes p}]$
- Approximate $\mathbb{P}$ by its **high order moments**

High Order Moments estimation

Recall:

$$\langle \underbrace{\mathbf{x} \otimes \cdots \otimes \mathbf{x}}_{\text{K times}}, \mathbf{y} \otimes \cdots \otimes \mathbf{y} \rangle = \langle \mathbf{x}, \mathbf{y} \rangle^K$$

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iid random MacLaurin projections $\mathbf{w}_k$ such that $\mathbb{E}[\mathbf{w}_k \mathbf{w}_k^\top] = I$:

$$\begin{aligned}
\mathbb{E}[\langle \mathbf{x}, \mathbf{w}_1 \rangle \dots \langle \mathbf{x}, \mathbf{w}_K \rangle \langle \mathbf{y}, \mathbf{w}_1 \rangle \dots \langle \mathbf{y}, \mathbf{w}_K \rangle] &= \mathbb{E}[\langle \mathbf{x}, \mathbf{w}_1 \rangle \langle \mathbf{y}, \mathbf{w}_1 \rangle] \dots \mathbb{E}[\langle \mathbf{x}, \mathbf{w}_K \rangle \langle \mathbf{y}, \mathbf{w}_K \rangle] \\
&= \mathbf{x}^\top \mathbb{E}[\mathbf{w}_1 \mathbf{w}_1^\top] \mathbf{x} \dots \mathbf{y}^\top \mathbb{E}[\mathbf{w}_K \mathbf{w}_K^\top] \mathbf{y} \\
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Define explicit mapping ϕ:

$$\phi_K(\mathbf{x}) = \prod_{k=1}^{K} \langle \mathbf{w}_k \ ; \ \mathbf{x} \rangle$$

then:

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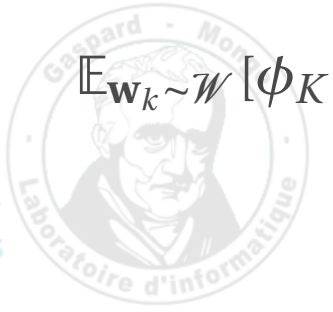

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$\Rightarrow$ **Untrainable projections $\mathbf{w}_k$**

Recursive HOM estimation[4]

Using trainable projections $\mathbf{W}_k$:

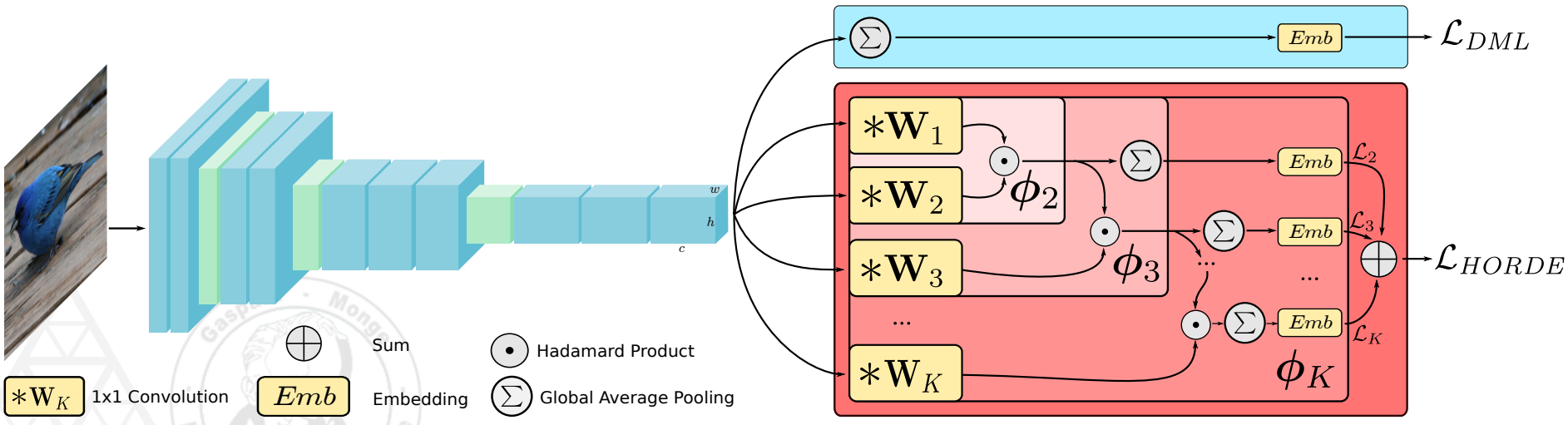
$$\boldsymbol{\phi}_k(\mathbf{x}) = \boldsymbol{\phi}_{k-1}(\mathbf{x}) \odot \left(\mathbf{W}_k^\top \mathbf{x}\right)$$

Average several projections:

$$\langle \mathbf{x} \ ; \ \mathbf{y} \rangle^K = \mathbb{E}[\prod \langle x, \mathbf{w}_k \rangle \langle y, \mathbf{w}_k \rangle] \approx \frac{1}{d} \langle \boldsymbol{\phi}_K(\mathbf{x}) \ ; \ \boldsymbol{\phi}_K(\mathbf{y}) \rangle$$

Cascade architecture

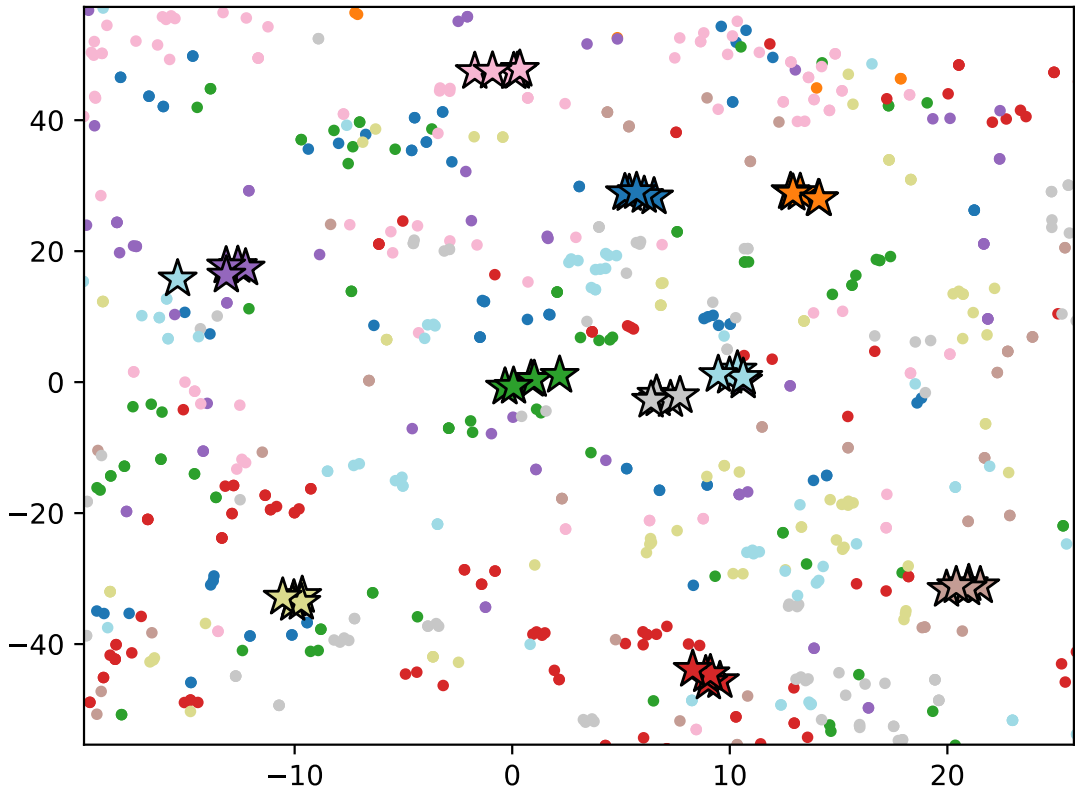
- Fewer weights prevents overfitting and computationally efficient



4 Jacob, Pierre, et al. "Metric learning with horde: High-order regularizer for deep embeddings." Proceedings of the IEEE International Conference on Computer Vision. 2019.

Qualitative results

Full sampling **without** HORDE regularization



MNIST, test data, ⋆ image representations, • deep features

Qualitative results

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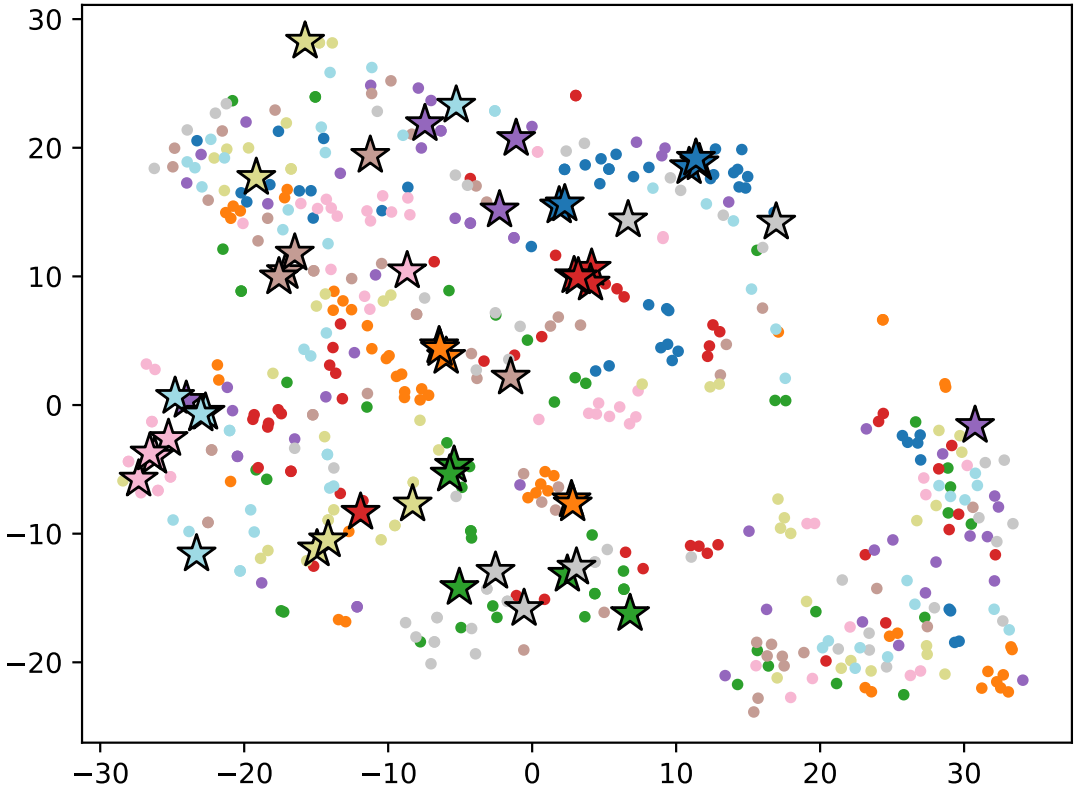

MNIST, test data, ⋆ image representations, • deep features

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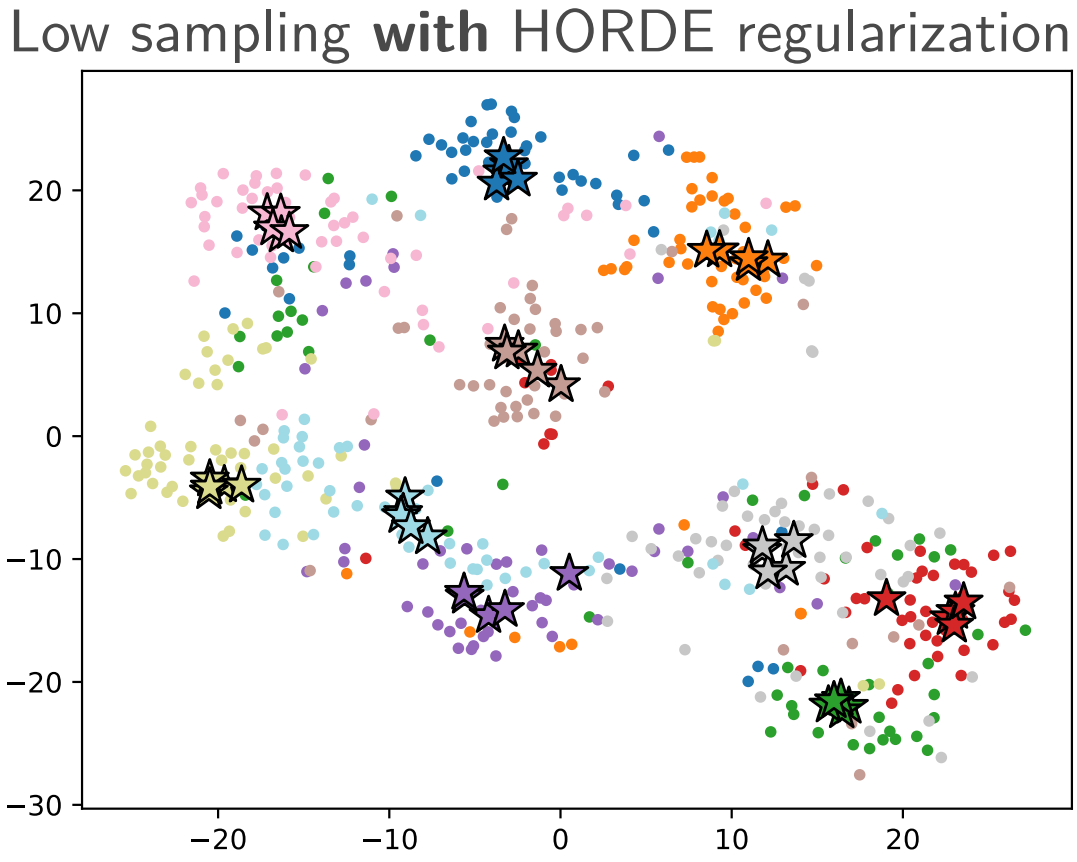


MNIST, test data, ⋆ image representations, • deep features

HO impact

R@1: Percentage nearest neighbor is correct

k	1	2		3			4				5					6					
n	1	1	2	1	2	3	1	2	3	4	1	2	3	4	5	1	2	3	4	5	6
R@1	55.9	**57.8**	58.6	56.8	58.0	56.9	**57.8**	58.8	57.6	56.1	57.4	57.7	56.8	56.3	53.3	57.4	57.9	57.1	55.6	54.4	50.7

MacLaurin

k	1	2		3			4				5					6					
n	1	1	2	1	2	3	1	2	3	4	1	2	3	4	5	1	2	3	4	5	6
R@1	55.9	57.0	53.4	57.6	54.7	50.6	57.9	55.4	52.3	47.6	58.1	55.9	53.1	48.4	43.7	**58.4**	55.7	52.9	47.8	43.9	40.5

Full train

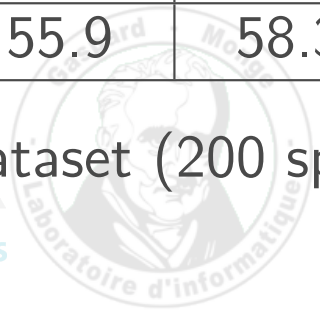
k	1	2		3			4				5					6					
n	1	1	2	1	2	3	1	2	3	4	1	2	3	4	5	1	2	3	4	5	6
R@1	55.9	57.0	53.4	57.9	56.1	54.2	57.6	55.4	54.3	53.0	**58.3**	56.3	56.0	54.7	52.4	57.9	56.6	55.8	55.0	53.9	51.6

HORDE

Concatenation of all HO:

	Baseline	HORDE	HORDE concat
R@1	55.9	58.3	**59.4**

CUB dataset (200 species of birds)



Conclusion

Image similarity based on local patterns:

- Matching keypoints is ++ but non ML/DL compatible
- Matching kernels are ML compatible drop in replacement
- Kernel linearization makes matching kernels DL compatible
- Local features matching rel. to matching distributions
- Matching distribution with MMD eq. to Kernel Mean Embedding
- High order moments rel. to Matching Kernel approx and KME

High order moments: embedding local information

Papers + Code

Contact: jacobpie@fel.cvut.cz (or david.picard@enpc.fr)

HORDE (2019):
Github: https://github.com/pierre-jacob/ICCV2019-Horde
Paper: https://openaccess.thecvf.com/content_ICCV_2019/html/Jacob_Metric_Learning_With_HORDE_High-Order_Regularizer_for_Deep_Embeddings_ICCV_2019_paper.html

JCF (2018):
Paper: https://arxiv.org/abs/1906.01972

STA (2016):
Paper: https://hal.archives-ouvertes.fr/hal-01359109

VLAT (2011):
Paper: https://hal.archives-ouvertes.fr/hal-00591993/